THE TEMPERATURE FIELD IN A HOLLOW CYLINDER DUE TO A SOURCE MOVING ALONG A HELIX

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We solve the problem of the temperature distribution in a hollow cylinder due to a source moving along a helix on the outer surface with boundary conditions of the second kind. The solution can be used to calculate temperature fields on a computer for various forms of processing materials.

A heat source of length $2h_0$ and (angular) width 2β moves along a helix on the outer surface of an infinitely long hollow cylinder, starting at some time t = 0. The source intensity q(t) varies with the time in some way. The initial cylinder temperature is everywhere T_0. There is no heat transfer to the external medium. The thermophysical parameters are assumed to be constant.

It is required to establish the temperature distribution for t > 0.

We shall solve the problem in cylindrical coordinates moving with the source. The origin is on the cylinder axis and the initial plane from which the angle φ is measured passes through the middle of the source.

Then the problem can be reduced to the solution of the following equation (in nondimensional variables):

$$\frac{\partial\Theta}{\partial \mathrm{Fo}} = \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Theta}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Theta}{\partial\varphi^2} + \frac{\partial^2\Theta}{\partial\xi^2} + U\frac{\partial\Theta}{\partial\xi} + \Omega\frac{\partial\Theta}{\partial\varphi} \quad (1)$$

with initial and boundary conditions

$$\Theta(\rho,\ \xi,\ \varphi,\ 0) = 0,$$

$$\left.\frac{\partial\Theta}{\partial\rho}\right|_{\rho=1} = \begin{cases} 0 & |\xi| > h, \\ \left.\begin{array}{ll} 0 & |\varphi| > \beta \\ K(\mathrm{Fo}) & |\varphi| \leqslant \beta \end{array}\right\} & |\xi| \leqslant h, \end{cases}$$

$$\left.\frac{\partial\Theta}{\partial\rho}\right|_{\rho=\rho_0} = 0, \quad \rho_0 = \frac{R_1}{R_2}.$$

Equation (1) can be solved by the sequential application of Fourier and Hankel transforms [1-3].

Omitting the intermediate calculations, we can write the solution as

$$\Theta(\rho,\ \xi,\ \varphi,\ \mathrm{Fo}) = \frac{1}{\pi}\sum_{n=0}^{\infty}\beta_n\sum_{s}F_{ns}G_n(s\rho)\int_0^{\mathrm{Fo}}K(\mathrm{Fo}-\tau)\cos n(\varphi+\Omega\tau)\exp(-s^2\tau)\,\Phi(\xi,\ \tau)\,d\tau,$$

where

$$\beta_n = \delta_n\frac{\sin n\beta}{n\beta},$$

$$\delta_n = \begin{cases} \frac{1}{2} & n = 0, \\ 1 & n = 1,\ 2,\ 3,\ \ldots, \end{cases}$$

$$G_n = J_n(s\rho)Y_n'(s\rho_0) - Y_n(s\rho)J_n'(s\rho_0),$$

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$$F_{ns} = \frac{2s^2 G_n(s)}{(s^2 - n^2)\, G_n^2(s) - (s^2\rho_0^2 - n^2)\, G_n^2(s\rho_0)},$$

$$\Phi(\xi,\ \tau) = \operatorname{erf}\left(\frac{h + \xi + U\tau}{2\sqrt{\tau}}\right) + \operatorname{erf}\left(\frac{h - \xi - U\tau}{2\sqrt{\tau}}\right).$$

The sum with respect to s is over all positive roots of the equation

$$G_n'(s) = 0.$$

In particular cases:

1) when the angular velocity $\omega = 0$ ($\Omega = 0$) we obtain the solution for a source moving along the z-axis over the surface of the cylinder;

2) when the linear velocity $v = 0$ ($U = 0$) we obtain the solution for a source rotating about the axis of the cylinder;

3) when $\beta = \pi$ we obtain the solution for an annular source in motion;

4) when $\omega = v = 0$ ($\Omega = U = 0$) we obtain solutions for fixed sources;

5) when $\beta = \pi$ and $h = \infty$ we obtain the solution for a cylinder heated over the whole of its outer surface. If $K(Fo) = K = \text{const}$ in this case we obtain the solution given in [1] for boundary conditions of the second kind.

NOTATION

$\theta = T - T_0/T_0$	is the nondimensional temperature;
T	is the cylinder temperature;
T_0	is the initial cylinder temperature;
$\rho = r/R_2$	is the nondimensional radius;
R_1, R_2	are internal and external radii of the cylindrical surfaces;
$\xi = z/R_2$	is the nondimensional coordinate in the direction of the z-axis;
$Fo = (a/R_2^2)t$	is the Fourier number;
a	is the coefficient of thermal diffusivity;
λ	is the thermal conductivity coefficient;
$h = h_0/R_2$	is the nondimensional source length;
$K(Fo) = (R_2/\lambda T_0)q(Fo)$	is the nondimensional intensity of the heat flow;
$J_n(s\rho)$, $Y_n(s\rho)$	is the Bessel function;
$U = (v/a)R_2$	is the nondimensional velocity along z-axis;
$\Omega = (\omega/a)R_2^2$	is the nondimensional angular velocity.

LITERATURE CITED

1. A. V. Lykov, The Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
2. D. S. Kuznetsov, Special Functions [in Russian], Vysshaya Shkola, Moscow (1962).
3. P. I. Romanovskii, Fourier Series, Field Theory [in Russian], GITTL, Moscow (1957).